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LETTER TO THE EDITOR

Possible existence of quark stars

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Abstract. The problem of existence of a family of quark stars has been investigated using the ground-state equation of state for the quark gas in second-order perturbation theory in quantum chromodynamics.

With the discovery of neutrons it was pointed out by Landau (1932) that a super-dense system with a central density much higher than that obtained in a white dwarf may exist. It is now well known that dense systems called neutron stars do exist with a central density as high as 10^{15} g cm^{-3}. There naturally arises the question of what state of matter would occur if the central density increased beyond 10^{15} g cm^{-3}. With the present understanding and accumulation of experimental evidence it is now believed that hadrons are constituted of confined point-like objects called quarks. At very high densities the hadrons may come very close to each other so that they overlap, and as a result the constituent quarks do not recognise to which particular hadron they belong, leading to a super-dense system forming a quark soup. Collins and Perry (1975) considered the possibility of whether the core of the dense star could exist in this quark-matter phase. They showed that if the quarks interact via massless coloured octet gluons obeying a $SU_c(3)$ gauge theory, then at $T = 0$ the core could exist in the quark-matter phase with a density as high as 10^{16} g cm^{-3}. There has been a number of papers following this work. An important question which immediately arises is whether a hadron–quark phase transition is possible. This question has been investigated by several authors (Baym and Chin 1976, Chapline and Nauenberg 1976, 1977, Baluni 1978a, b, Brown and Rho 1977). In this Letter we are interested in investigating the possibility of the existence of a super-dense star with a core of quark matter. This was originally considered by Itoh (1970), assuming that the quarks are free and that they obey parastatistics, as was originally proposed to obtain the quark structure of the low-lying baryons.

Whether a third family of super-dense quarkion stars with a central density of 10^{16} g cm^{-3} and beyond could be a stable system or not has previously been considered by Bowers *et al* (1977), Keister and Kisslinger (1976) and Królak *et al* (1978). We will consider the quarks as a strongly interacting system at $T = 0$ and the interaction dynamics as that given by the quantum chromodynamics (QCD) obeying $SU_c(3)$. In this Letter we will first obtain a mass–radius relation of the super-dense quarkion system, assuming an equal amount of the three different quark flavours, for various possibilities of the central densities of the system.

In order to investigate the stability or otherwise of the super-dense configurations, we integrate numerically the relativistic hydrostatic equilibrium equation of

Oppenheimer and Volkopf, namely

$$dP/dR = -(G/c^4)(P + \rho c^2)(Mc^2 + 4\pi R^3 P)(R^2 - 2GMR/c^2)^{-1},$$
$$dM/dR = 4\pi R^2 \rho, \tag{1}$$

from a given value of the central density $\rho = \rho_0$ at the centre to the point of vanishing pressure at the surface of the dense system. In doing this we use the quantum chromodynamical equation of state for quark matter up to a point where the value of the density equals a certain critical density ρ_c, at which a first-order phase transition between neutron matter and quark matter is possible (Chapline and Nauenberg 1977); hence below this density we use two different forms of neutron-matter equations of state to represent the state of matter.

Now, for the quark system the ground-state equation of state for the quark gas in second-order perturbation theory in QCD, including the effect of the renormalisation of the quark–gluon coupling constant α_c, is given (Chapline and Nauenberg 1977) by

$$P = \frac{3}{4\pi^2}\Lambda_F^4\chi^4\left[1 + \frac{4}{27\ln\chi}\left(1 - \frac{1}{\ln\chi}\right)\right],$$
$$\rho = \frac{9}{4\pi^2}\Lambda_F^4\chi^4\left(1 + \frac{4}{27\ln\chi}\right), \tag{2}$$

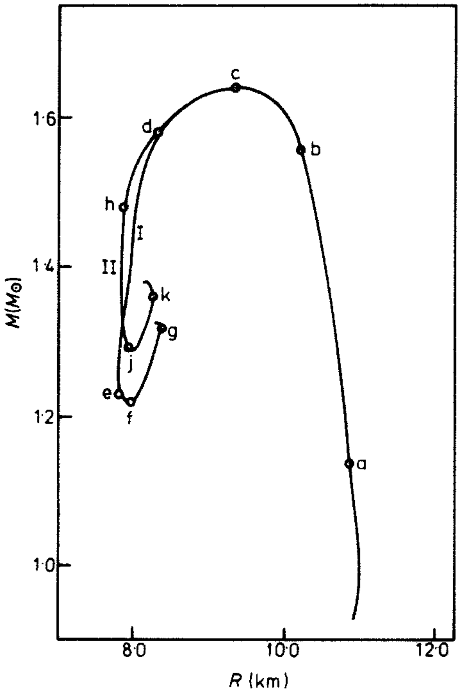

Figure 1. Mass (M) plotted against radius (R) by matching the quark-matter equation of state to the Bethe–Johnson VH equation of state for neutron matter. The curves I and II refer to $\Lambda_F = 300$ MeV and $\Lambda_F = 400$ MeV respectively, for quark matter (see text). The curves are parametrised by the central density in units of g cm^{-3}: a, $1{\cdot}095\times10^{15}$; b, $1{\cdot}758\times10^{15}$; c, $3{\cdot}307\times10^{15}$; d, $1{\cdot}028\times10^{16}$; e, $9{\cdot}08\times10^{16}$; d, $1{\cdot}187\times10^{17}$; g, $1{\cdot}961\times10^{18}$; h, $2{\cdot}6\times10^{16}$; j, $1{\cdot}234\times10^{17}$; k, $1{\cdot}884\times10^{18}$.

where $\hbar = c = 1$ is understood. In obtaining equation (2) we have used the following value for the subtraction-dependent chromo-fine structure constant α_c in a three-flavour quark system, namely

$$\alpha_c = \pi/18 \ln \chi,$$

where the dimensionless parameter $\chi = k_F/\Lambda_F$; k_F stands for the quark Fermi momentum, Λ_F is the subtraction constant, and quarks are taken to be massless.

For the low-density hadronic system we use the equations of state given by two representative models of hadronic matter, namely, the VH model of Bethe and Johnson (Malone *et al* 1975) and the neutron-solid model of Pandharipande and Smith (1975). For densities less than $1{\cdot}6 \times 10^{14}$ g cm^{-3} we use a combination of various equations of state as listed in detail by Baym *et al* (1971).

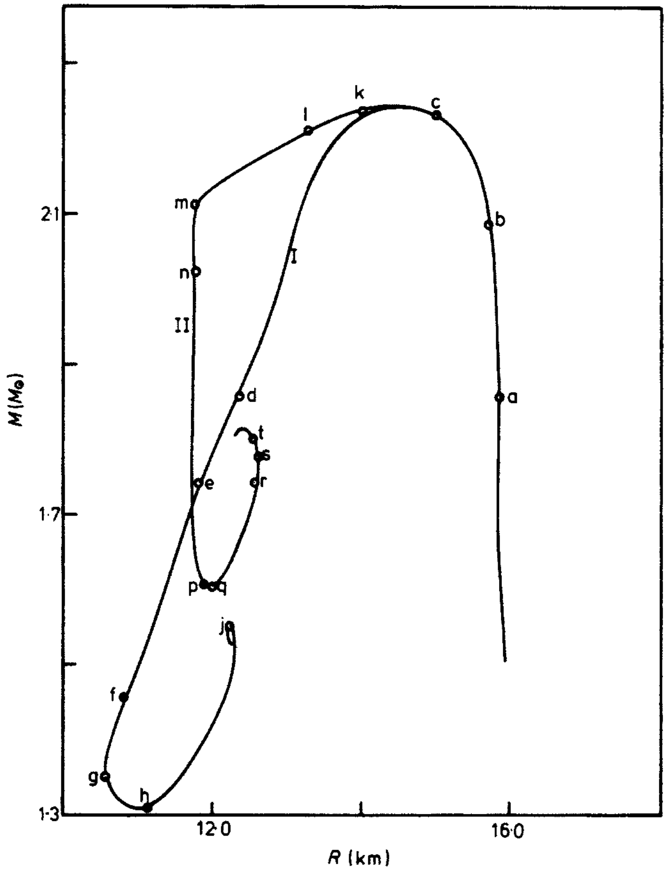

Figure 2. Mass plotted against radius by matching the quark-matter equation of state to the Pandharipande–Smith equation of state for neutron matter. The curves I and II refer to $\Lambda_F = 300$ MeV and $\Lambda_F = 400$ MeV respectively, for quark matter (see text). The curves are parametrised by the central density in units of g cm^{-3}: a, $4{\cdot}97 \times 10^{14}$; b, $6{\cdot}04 \times 10^{14}$; c, $1{\cdot}01 \times 10^{15}$; d, $6{\cdot}42 \times 10^{15}$; e, $8{\cdot}27 \times 10^{17}$; f, $2{\cdot}03 \times 10^{16}$; g, $3{\cdot}77 \times 10^{16}$; h, $1{\cdot}19 \times 10^{17}$; j, $1{\cdot}20 \times 10^{18}$; k, $1{\cdot}43 \times 10^{15}$; l, $1{\cdot}95 \times 10^{15}$; m, $9{\cdot}93 \times 10^{15}$; n, $1{\cdot}25 \times 10^{16}$; p, $8{\cdot}02 \times 10^{16}$; q, $1{\cdot}05 \times 10^{17}$; r, $6{\cdot}75 \times 10^{17}$; s, $1{\cdot}06 \times 10^{18}$; t, $1{\cdot}80 \times 10^{18}$.

Using equations (2) for quark matter in conjunction with the various hadronic-matter equations of state as specified above, we now solve equations (1) for the mass–radius relation of the equilibrium configurations. We have used two different values of the subtraction constant Λ_F, namely, 300 and 400 MeV. The curve I in figure 1 corresponds to the mass–radius relation for the quark stars, with equations (2) with $\Lambda_F = 300$ MeV matched to the Bethe–Johnson VH equation of state at the critical density $\rho_c = 5{\cdot}6 \times 10^{15}$ g cm^{-3}, while the curve II corresponds to the subtraction point at $\Lambda_F = 400$ MeV with the corresponding $\rho_c = 7{\cdot}39 \times 10^{15}$ g cm^{-3}. Figure 2 displays the mass–radius relation when the Bethe–Johnson model is replaced by the Pandharipande–Smith model for neutron matter. The curve I in figure 2 corresponds to $\Lambda_F = 300$ MeV with the critical density $\rho_c = 1{\cdot}8 \times 10^{15}$ g cm^{-3}, while curve II represents the choice $\Lambda_F = 400$ MeV with the corresponding $\rho_c = 3{\cdot}0 \times 10^{15}$ g cm^{-3}. The shape of the curves together with the criteria of stability given by Bardeen *et al* (1966) clearly indicate that the configurations containing quark matter at the core are unstable, as is also the case in the bag-model calculations of Królak *et al* (1978). The relation between the masses of the quark stars and the central densities is shown in figures 3 and 4. It appears that, irrespective of the choice of the values of Λ_F, the maximum mass of the quark star $\approx 1{\cdot}64$ $M_\odot$ (where $M_\odot$ is the solar mass) if one uses the Bethe–Johnson VH model for the neutron matter. Similarly, the Pandharipande–Smith model matched with the QCD equation of state for the quarks predicts a maximum mass $\approx 2{\cdot}24$ $M_\odot$. The present estimate of neutron-star masses can be as high as 3–5 $M_\odot$ (Baym and Pethick 1975). Thus the maximum quark-star masses appear to be less than the maximum masses of neutron stars. This shows that the masses greater than the maximum mass of the neutron stars are not permissible in the hydrostatic equilibrium even with higher central densities, and will undergo a gravitational collapse as pointed out by Itoh (1970). Baym and Chin (1976) have come to similar conclusions using Gerlach's analysis (Gerlach 1968). To obtain more realistic results we should take into account non-vanishing quark masses and also higher-order QCD corrections to obtain the expressions for the equation of state for quark matter. The results with higher-order QCD corrections and non-vanishing quark masses will be reported shortly.

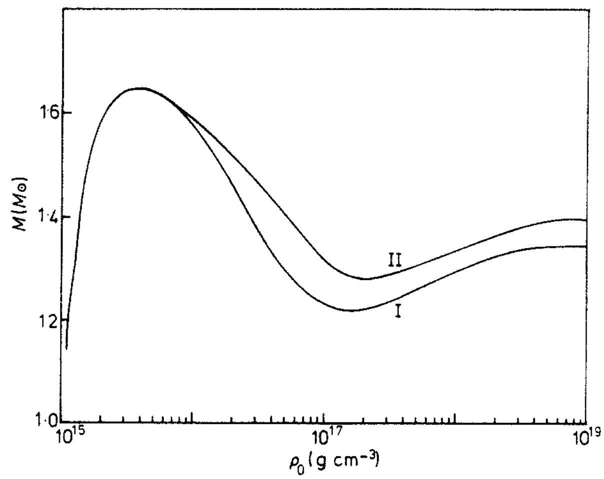

Figure 3. Mass (M) plotted against central density (ρ_0) for the Bethe–Johnson VH equation of state for neutron matter matched to the quark-matter equations of state, for Λ_F = 300 MeV (curve I) and $\Lambda_F = 400$ MeV (curve II).

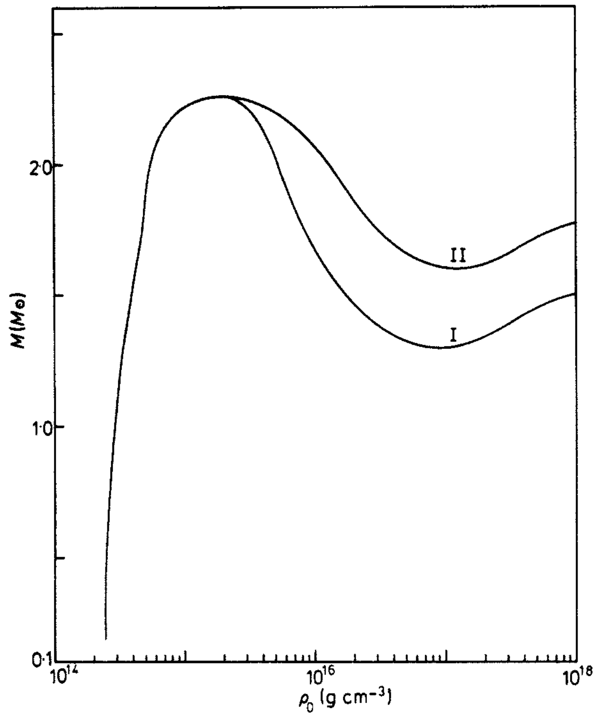

Figure 4. Mass plotted against central density for the Pandharipande–Smith equation of state for neutron matter matched to the quark-matter equations of state, for $\Lambda_F = 300$ MeV (curve I) and $\Lambda_F = 400$ MeV (curve II).

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